MATHEMATICS

I Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits (AL/3)
MTH 101	Calculus of One Variable	3	1	4.5	0	0	8.5	4	O to F	3

II Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits (AL/3)
MTH 102	Linear Algebra	3	1	4.5	0	0	8.5	4	O to F	3

III Semester

	Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits (AL/3)
Mathematics											
DC	MTH201	Multivariable Calculus	3	1	4.5	0	0	8.5	4	O to F	3
	MTH203	Introduction to Groups and Symmetry	3	1	4.5	0	0	8.5	4	O to F	3

IV Semester

	Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits (AL/3)
Mathematics											
DC	MTH202	Probability and Statistics	3	1	4.5	0	0	8.5	4	O to F	3
	MTH204	Complex Variables	3	1	4.5	0	0	8.5	4	O to F	3

DC: Departmental Compulsory Course

V Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits
MTH 301	Group Theory	3	0	7.5	0	0	10.5	3	O to F	4
MTH 303	Real Analysis I	3	0	7.5	0	0	10.5	3	O to F	4
MTH 305	Elementary Number Theory	3	0	7.5	0	0	10.5	3	O to F	4
MTH ***	Departmental Elective I	3	0	7.5	0	0	10.5	3	O to F	4
*** ***	Open Elective I	3	0	4.5/7.5	0	0	7.5/10.5	3	O to F	3/4
Total Credits		**15**	**0**	**34.5/37.5**	**0**	**0**	**49.5/52.5**	**15**		**19/20**

VI Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits
MTH 302	Rings and Modules	3	0	7.5	0	0	10.5	3	O to F	4
MTH 304	General Topology	3	0	7.5	0	0	10.5	3	O to F	4
MTH 306	Ordinary Differential Equations	3	0	7.5	0	0	10.5	3	O to F	4
MTH ***	Departmental Elective II	3	0	7.5	0	0	10.5	3	O to F	4
*** ***	Open Elective II	3	0	4.5/7.5	0	0	7.5/10.5	3	O to F	3/4
Total Credits		**15**	**0**	**34.5/37.5**	**0**	**0**	**49.5/52.5**	**15**		**19/20**

VII Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits
MTH 401	Fields and Galois Theory	3	0	7.5	0	0	10.5	3	O to F	4
MTH 403	Real Analysis II	3	0	7.5	0	0	10.5	3	O to F	4
MTH 405	Partial Differential Equations	3	0	7.5	0	0	10.5	3	O to F	4
MTH 407	Complex Analysis I	3	0	7.5	0	0	10.5	3	O to F	4
*** ***	Open Elective III	3	0	4.5/7.5	0	0	7.5/10.5	3	O to F	3/4
Total Credits		**15**	**0**	**34.5/37.5**	**0**	**0**	**49.5/52.5**	**15**		**19/20**

VIII Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits
MTH 404	Measure and Integration	3	0	7.5	0	0	10.5	3	O to F	4
MTH 406	Differential Geometry of Curves and Surfaces	3	0	7.5	0	0	10.5	3	O to F	4
MTH ***	Departmental Elective III	3	0	7.5	0	0	10.5	3	O to F	4
*** ***	Open Elective IV	3	0	4.5/7.5	0	0	7.5/10.5	3	O to F	3/4
*** ***	Open Elective V	3	0	4.5/7.5	0	0	7.5/10.5	3	O to F	3/4
Total Credits		**15**	**0**	**31.5/37.5**	**0**	**0**	**46.5/52.5**	**15**		**18/20**

IX Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits
MTH 501	MS Thesis	-	-	-	-	-	30	-	O to F	12
MTH 503	Functional Analysis	3	0	7.5	0	0	10.5	3	O to F	4
MTH ***	Departmental Elective V	3	0	7.5	0	0	10.5	3	O to F	4
HSS 503	Law Relating to Intellectual Property and Patents	1	0	2.5	0	0	3.5	1	S/X	1
Total Credits		**7**	**0**	**17.5**	**0**	**0**	**54.5**	**7**		**21**

X Semester

Course No.	Course Title	Lec Hr	Tut Hr	SS Hr	Lab Hr	DS Hr	AL	TC Hr	Grading System	Credits
MTH 501	MS Thesis	-	-	-	-	-	30	-	O to F	12
MTH ***	Departmental Elective VI	3	0	7.5	0	0	10.5	3	O to F	4
MTH ***	Departmental Elective VII	3	0	7.5	0	0	10.5	3	O to F	4
Total Credits		**6**	**0**	**15**	**0**	**0**	**51**	**6**		**20**

MTH 101: Calculus of One Variable (3)

Learning Objectives

This is a core mathematics course for first-semester BS-MS students. The course introduces the basic concepts of differential and integral calculus of one real variable with an emphasis on careful reasoning and understanding of the material.

Course Contents

- *Introduction to the real number system*: algebraic and order properties, bounded sets, supremum and infimum, completeness property, integers and rationals, absolute value and triangle inequality
- *Sequences and series*: convergence of a sequence, Cauchy's criterion, limit of a sequence, supremum and infimum, absolute and conditional convergence of an infinite series, tests of convergence, examples
- *Limits and continuity*: definitions, continuity and discontinuity of a function at a point, left and right continuity, examples of continuous and discontinuous functions, intermediate value theorem, boundedness of a continuous function on a closed interval, uniform continuity
- *Differentiation*: definition and basic properties, Rolle's theorem, mean value theorem, Leibnitz's theorem on successive differentiation, Taylor's theorem
- *Integration*: Riemann integral viewed as an area, partitions, upper and lower integrals, basic properties of the Riemann integral, fundamental theorem of calculus, integration by parts, applications.

Suggested Books

1. G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry*, 9th edition, Indian student edition, Addison-Wesley, 1998
2. G. B. Thomas, M. D. Weir, J. R. Hass, *Thomas' Calculus*, 12th Edition, Pearson, 2014.
3. T. M. Apostol, *Calculus*, Volumes 1 and 2, 2nd edition, Wiley Eastern, 1980
4. J. Stewart, *Calculus: Early Transcendentals*, Cengage Learning

MTH 102: Linear Algebra (3)

Learning Objectives

This course focuses on the elementary theory of matrices. Most of the concepts in this theory find their origins in a systematic study of solutions of a given set of finitely many linear equations in finitely many unknowns. Soon enough in this study, one realizes that there is a broader mathematical framework in which all of these concepts can be suitably defined and studied, with the results obtained therein having a greater value. We introduce and study the rudiments of this framework of vector spaces, and linear operators between any two of them. This course ends with a discussion of the very important spectral theorem for symmetric matrices.

Course Contents

- *Matrices*: Review of complex numbers, matrix operations, special matrices (diagonal, triangular, symmetric, skew-symmetric, orthogonal, hermitian, skew hermitian, unitary, normal), vectors in R^n and C^n, matrix equation Ax = b, row-reduced echeln form, row space, column space, and rank of a matrix, determinants, systems of linear equations.
- *The space* R^n: Linear independence and dependence, linear span, linear subspaces.
- *Finite-dimensional vector spaces*: Bases and dimensions, linear transformations, matrix of a linear transformation, Rank-nullity theorem.
- *Inner product spaces*: Orthonormal bases, Gram-Schmidt orthogonalization, projections.
- *Linear operators*: Eigenvalues and eigenvectors of a linear operator, characteristic polynomial, diagonalizability of a linear operator, eigenvalues of the special matrices stated above, Spectral theorem for real symmetric matrices, and its application to quadratic forms, positive definite matrices.

Suggested Books

1. H. Anton, *Elementary linear algebra and applications,* 8th edition, John Wiley, 1995.
2. D. C. Lay, *Linear algebra and its applications*, 3rd Edition, Pearson, 2011.

3. G. Strang, *Linear algebra and its applications,* 4th edition, Thomson, 2006.
4. S. Lang, *Linear Algebra,* 3rd Edition, Springer, 1987.
5. S. Kumaresan, *Linear algebra - A Geometric Approach,* Prentice Hall of India, 2000.

MTH 201: Multivariable Calculus (3)

Learning Objectives

This course generalizes the various concepts and results pertaining to function of one real variable to functions of several real variables. The material covered in this course is used extensively in physical and engineering sciences. It also lays the foundation for courses such as Real Analysis II and Differential Geometry of Curves and Surfaces, which form an integral part of the mathematics core curriculum

Course Contents

- *Vector algebra*: Vectors in R^3, dot product of vectors, length of a vector, orthogonality of vectors, cross product of vectors.
- *Geometry in R^3*: Lines, planes, and quadric surfaces.
- *Vector-valued functions*: Continuity and differentiability of vector-valued functions of real variable, curves in R^3, tangent vectors.
- *Multivariable functions*: Limits and continuity, partial derivatives, gradient, directional derivatives, maxima, minima and saddle points, Lagrange multipliers.
- *Integration*: Double and triple integrals, change of coordinates, vector fields, line integrals, surface integrals, statements of the Green's, Divergence, and Stokes' theorems, and their applications.

Suggested Books

1. G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry,* 9th edition, Indian student edition, Addison-Wesley, 1998
2. G. B. Thomas, M. D. Weir, J. R. Hass, *Thomas' Calculus*, 12th Edition, Pearson, 2014.

3. J. Stewart, *Calculus*, 7^{th} Edition, Cengage Learning, 2012.
4. J. E. Marsden and A. Tromba, *Vector Calculus*, W.H. Freeman & Company, 2004

MTH 202: Probability and Statistics (3)

Learning Objectives

This course aims at providing an introduction to the theory of probability and statistics. No prior knowledge in this subject is required. However, basic knowledge in Calculus and Linear Algebra would be required. The course will cover basic probability, probability distributions, random variables, sampling distributions, and statistical inference (estimation and hypothesis testing).

Course Contents

- *Probability*: Classical, relative frequency and axiomatic definitions of probability, addition rule and conditional probability, multiplication rule, total probability, Bayes Theorem and independence, problems
- *Combinatorial analysis*: Permutations, Combinations, partitions
- *Random Variables*: Discrete, continuous and mixed random variables, probability mass, probability density and cumulative distribution functions, mathematical expectation, moments, probability and moment generating function, median and quantiles, Markov inequality, Chebyshev’s inequality, problems
- *Special Distributions, Joint Distributions*: Joint, marginal and conditional distributions, product moments, independence of random variables, bivariate normal distribution
- *Multivariate distributions*: Properties, distributions of sums and quotients of random variables
- *Sampling Distributions*: The Central Limit Theorem, distributions of the sample mean and the sample variance for a normal population, Chi-Square, t and F distributions
- *Descriptive Statistics*: Graphical representation, Summarization and tabulation of data

- *Estimation*: Unbiasedness, consistency, the method of moments and the method of maximum likelihood estimation, confidence intervals for parameters in one sample and two sample problems of normal populations, confidence intervals for proportions
- *Testing of Hypotheses*: Null and alternative hypotheses, the critical and acceptance regions, two types of error, power of the test, Neyman-Pearson Fundamental Lemma, tests for one sample and two sample problems for normal populations, tests for proportions, Chi-square goodness of fit test and its applications

Suggested Textbooks

1. P. Hoel, S. Port, C. Stone, *Introduction to Probability Theory*, 1st Edition, Brooks Cole, 1972
2. P. Hoel, *Introduction to Mathematical Statistics,* 5th Edition, Wiley, 1984
3. V. Rohatgi, A. Saleh, *Introduction to Probability Theory and Statistics,* 2nd Edition, Wiley, 2000

References

1. Richard Isaac, *The Pleasures of Probability,* Undergraduate Texts in Mathematics, Springer, 1995
2. J. Schiller, R. Srinivasan, M. Spiegel, *Schaum's Outline of Probability and Statistics,* 4th Edition, McGraw Hill Education, 2013
3. W.Feller, *An Introduction to Probability Theory and Its Applications,* Volume 1, 3rd Edition, Wiley, 1968
4. S.M. Ross, *A First Course in Probability,* 6th Edition, Prentice Hall

MTH 203: Introduction to Groups and Symmetry (3)

Learning Objectives

Symmetries in nature are a source of curiosity for various scientific fields. Often these are the reasons of stability of various structures and patterns formed in nature. The study of symmetries is naturally intertwined with the concept of transformations of the corresponding objects. Group theory arose in nineteenth century to formalize these ideas. This course is aimed at building these ideas as we explicitly try to understand the nature of symmetries that occur in each individual case and compute these in details.

Course Contents

- Examples of symmetries: Symmetries of equilateral triangle and square; translations, rotations and reflections in the Euclidean plane.
- Definition of a group, subgroup, abelian group, group Z of integers, statement of division algorithm, description of all subgroups of Z.
- Equivalence relations, group of congruence classes Zn, order of an element in a group, definition of cyclic group, cyclicity of groups of prime order, group of units in Zn.
- Definition of a homomorphism and normal subgroup, kernel and image of a homomorphism, quotient group, isomorphism theorems (statement and applications).
- Permutations of a finite set, permutation group S_n, cycle notation, length of a cycle, transpositions, decomposing a permutations as a product of transpositions, parity of a permutation, alternating group An as normal subgroup, conjugacy in permutation groups, generating sets of S_n and A_n.
- Groups of real and complex matrices: general linear groups, determinant of a matrix as a group homomorphism, special linear groups, complex matrices as real matrix, orthogonal and special orthogonal groups, unitary and special unitary groups.
- Two dimensional symmetries: group of symmetries of geometric objects in Euclidean spaces, dihedral group as the group of symmetries of a regular polygon, isometries of the Euclidean plane, a detailed account of the classification of isometries: translations, rotations, reflections, glide reflections; wallpaper symmetries, finite subgroups of SO(2).
- Three dimensional symmetries: platonic solids and their dual, symmetries of a tetrahedron, symmetries of a cube and octahedron, symmetries of icosahedron and dodecahedron, classification of finite subgroups of SO(3).

Suggested Books

1. Online notes: https://neil-strickland.staff.shef.ac.uk/courses/groups/
2. Online Notes Groups and Symmetry, Andrew Baker http://www.maths.gla.ac.uk/~ajb/dvi-ps/2q-notes.pdf

3. Groups and Symmetry (Undergraduate Texts in Mathematics), Mark A. Armstrong, Springer, 1997
4. A First Course in Abstract Algebra (3rd Edition), Joseph J. Rotman, Pearson, 2005
5. Algebra (2nd Edition), M. Artin, Pearson, 2010
6. Matrix groups for Undergraduates, Kristopher Tapp, AMS, 2005
7. Symmetry: A Mathematical Exploration, Kristopher Tapp, Springer, 2012
8. Online notes: http://www.math.columbia.edu/~bayer/F03/symmetry/

MTH 204: Complex Variables (3)

Learning Objectives

By extending the real number system to include $\sqrt{-1}$, one obtains the set of complex numbers, which possesses an algebraic structure similar to real numbers. This motivates us to study the calculus of functions of a complex variable. Though a generalization, this calculus has a striking difference from the calculus of a real variable, which leads to some surprising results. The course intends to highlight some of these important results. This course is a precursor to MTH 407 Complex Analysis-I and the material covered is also used widely in physical and engineering sciences.

Course Contents

- Review of complex numbers, functions of one complex variable, limits and continuity, definition and examples of analytic functions, Cauchy-Riemann equations, definition of a harmonic function, harmonic conjugates
- Representation of an analytic function as a power series, term by term differentiation, elementary complex functions and comparison with real counterparts
- Contour integration, statement of Goursat’s theorem, proof of Cauchy’s theorem in a disc, Cauchy’s integral formulae
- Zero set of an analytic function, form of an analytic function in a neighborhood of a zero, definition and examples of removable singularities, poles, essential singularities respectively, Laurent series expansion of a complex function

- Residues, residue theorem in a disc, evaluation of real integrals and improper integrals

Suggested Books

1. R. V. Churchill and J. W. Brown, *Complex variables and applications* (7th edition), McGraw-Hill, 2003
2. J. M. Howie, *Complex Analysis*, Springer-Verlag, 2004
3. E.M. Stein and R. Shakarchi, Complex Analysis, Overseas Press (India) Pvt. Ltd. 2006
4. Murray R. Spiegel, Theory and Problems of Complex Variables, Schaum's Outline Series (McGraw-Hill), 2009

MTH 301: Group Theory (4)

Learning Objectives

This is an introductory course on Group theory. We will begin by studying the basic concepts of subgroups, homomorphisms and quotient groups with many examples. We then study group actions, and prove the Class equation and the Sylow theorems. They are in turn used to prove the structure theorem for finite abelian groups and to discuss the classification of groups of small order. We then turn to solvability, prove the Jordan-Holder theorem, and discuss nilpotent groups (if time permits).

Course Contents

- Definition of group, basic properties, examples (Dihedral, Symmetric, Groups of Matrices, Quaternion Group, Cyclic, Abelian Groups)
- Homomorphisms, Isomorphisms, subgroups, subgroup generated by a set, subgroups of cyclic groups
- Review of Equivalence relations, Cosets, Lagrange's theorem, Normal subgroup, Quotient Group, Examples, Isomorphism theorems, Automorphisms
- Group actions, orbits, stabilizer, faithful and transitive actions, centralizer, normalizer, Cayley's theorem, Action of the group on cosets
- Conjugation, Class equation, Cauchy's theorem, Applications to p-groups, Conjugacy in S_n

- Sylow theorems, Simplicity of A_n and other applications
- Direct products, Structure of Finite abelian groups
- Semi-Direct products, Classification of groups of small order
- Normal series, Composition series, Solvable groups, Jordan-Hölder theorem, Insolvability of S_5
- Lower and upper central series, Nilpotent groups, Basic commutator identities, Decomposition theorem of finite nilpotent groups (if time permits)

Suggested Books

1. I. N. Herstein, Topics in Algebra, 2nd Edition, Wiley, 2006
2. T. W. Hungerford, Algebra, Springer Verlag, 2005
3. M. Artin, Algebra, Prentice-Hall of India, 1994
4. D. S. Dummit, R. M. Foote, Abstract Algebra, 2nd Edition, Wiley
5. J. Rotman, A First Course in Abstract Algebra : With Applications, Prentice Hall
6. J. Rotman, An Introduction to Theory of Groups, Springer GTM, 1999
7. H. Kurzweil, B. Stellmacher, The Theory of Finite Groups, Springer Universitext, 2004
8. M. Suzuki, Group Theory I, Springer GMW 247

MTH 302: Rings and Modules (4)

Pre-requisites: *MTH 301*

Learning Objectives

This is an introductory course on Ring theory and Modules. We begin with the basic definitions and examples of rings, and discuss ideals, quotient rings, and the Chinese remainder theorem. We then discuss the important classes of commutative rings, irreducibility in general and specifically in the context of polynomial rings. We then introduce modules and some basic notions before discussing generating sets and free modules. We then prove the structure theorem for finitely generated modules over a PID and its applications. Finally, we discuss tensor products and some homological algebra (if time permits).

Course Contents

- Definition of rings, Homomorphisms, basic examples (Polynomial ring, Matrix ring, Group ring), Integral domain, field, Field of fractions of an integral domain
- Ideals, Prime and Maximal ideals, Quotient Rings, Isomorphism theorems, Chinese Remainder theorem, Applications
- Principal ideal domains, Irreducible elements, Unique factorization domains, Euclidean domains, examples
- Polynomial rings, ideals in polynomial rings, Polynomial rings over fields, Gauss' Lemma, Polynomial rings over UFDs, Irreducibility criteria
- Definition of modules, submodules, The group of homomorphisms, Quotient modules, Isomorphism theorems, Direct sums, Generating set, free modules, Simple modules, vector spaces
- Free modules over a PID, Finitely generated modules over PIDs
- Applications to finitely generated abelian groups and Rational and Jordan canonical forms
- (if time permits) Tensor product of modules, Exact sequences of modules, Hom functor, Projective modules, Injective modules, Baer's criterion

Suggested Books

1. D.S. Dummit, R.M. Foote, Abstract Algebra, 2nd Edition, Wiley
2. G. Birkhoff, S. McLane, Algebra (3rd Edition), AMS
3. S. Lang, Algebra (3rd Edition), Pears
4. C. Musili, Rings and Modules (2nd Edition), Narosa
5. M.F. Atiyah, I.G. Macdonald, Introduction to Commutative Algebra (1st Indian Edition), Levant Books
6. N. Jacobson, Basic Algebra (Vols - I & II), Hindustan Book Agency

MTH 303: Real Analysis I (4)

Learning Objectives

This is an introductory course on analysis for BS-MS mathematics students. The aim of this course is to introduce and develop basic analytic concepts of limit, convergence, integration and differentiation.

Course Contents

- *Real numbers*: The algebraic and order properties of R, absolute value of real numbers, the supremum and infimum properties, completeness of R, the Archimedean property, intervals, open sets, closed sets, compact and connected sets, Cantor set.
- *Functions on R*: Limit, Continuity and differentiability of real-valued functions, monotone functions, uniformly continuous functions, continuity and compactness, continuity and connectedness, functions of bounded variation, total variation.
- *Sequences and series of functions*: Pointwise and uniform convergence of sequences and series of functions, uniform convergence and its consequences, space of continuous functions on a closed interval, equicontinuous families, Arzela-Ascoli theorem, Weierstrass approximation theorem.
- *Power series and special functions*: Taylor's theorem, power series, radius of convergence, exponential, trigonometric and logarithmic functions.
- *Riemann-Stieltjes integral*: Definition and properties of Riemann-Stieltjes integral, differentiation of the integral, fundamental theorem of calculus, integration by parts.

Suggested Books

1. W. Rudin, *Principles of Mathematical Analysis, 3rd edition,* McGraw Hill, 1953.
2. R. G. Bartle and D. R. Sherbert, *Introduction to real analysis*, 4th edition, Wiley, 2011.
3. T. M. Apostol, *Mathematical Analysis, 2nd edition,* Narosa Publishing, 1985.
4. R. R. Goldberg, *Methods of Real Analysis,* Oxford & Ibh, 2012.

MTH 304: General Topology (4)

Pre-requisites: *MTH 303Real Analysis I*

Learning Objectives

General topology or point-set topology, as it is otherwise known, lies at the cornerstone of almost all areas in modern mathematics. This course covers all the basic notions in topology, and prepares the student for advanced courses in analysis, geometry, and topology.

Course Contents

- *Topological spaces*: Topology on a set and open sets; Examples of topological spaces; Coarse and fine topologies; Basis and subbasis for a topology; Subspace topology; Closed sets and limit points.
- *Continuity*: Continuous maps between topological spaces; Properties of continuous maps; Open and closed maps; Homeomorphisms; Topological embedding; Pasting Lemma.
- Product topology. The product topology on $X\times Y$; The product and box topologies for arbitrary products; Projection maps; Properties of the product topology.
- *Metrizable spaces*: Metric on spaces; Uniform metric and topology; Metrizability of the product topology; Sequence Lemma; Sequential definition of continuity; Uniform limit Theorem.
- *Quotient topology*: Quotient maps; Open and closed maps; Saturated open sets; Quotient spaces with examples; Properties of quotient spaces.
- *Connectedness*: Connected and path connected spaces with examples; Connected and path components; Totally disconnected spaces;Locally connected spaces; Properties of connected and path connected spaces.
- *Compactness*: Open covers for spaces; Compact spaces; Tube lemma and compactness for finite products; Finite intersection property and the Tychonoff's theorem; Heine-Borel Theorem; Extreme value theorem; Lebesque number lemma; Uniform continuity theorem; Limit point compactness; Sequential compactness; Local compactness; One-point compactification.

- *Countability axioms*: First and second countable spaces with examples; Properties of first and second countable spaces; Dense subsets and separability; Lindelöf spaces.
- *Separation axioms*: T_1 and Hausdorff spaces with examples; Regular and Normal spaces with examples; Properties of Hausdorff, regular and normal spaces; Urysohn's Lemma; Completely regular spaces and their properties; Urysohn metrization theorem; Tietze's extension theorem.

Suggested Books

1. J. R. Munkres, *Topology (2nd Ed),* Dorling Kindersley, 2006.
2. C. Adams and R. Franzosa, *Introduction to topology*, Pearson
3. Prentice Hall, 2008.
4. G. F. Simmons, *Introduction to Topology and Modern Analysis,*
5. Tata McGraw Hill, 2008.
6. B. Mendelson, *Introduction of topology*, Dover, 1990.
7. M. A. Armstrong, *Basic topology*, Springer International, 1983.
8. R. A. Conover, *A first course in topology,* Dover, 2003.
9. S. Kumaresan, *Topology of metric spaces (2nd Ed),* Narosa, 2011.

MTH 305: Elementary Number Theory (4)

Pre-requisites (recommended)*: MTH 101: Calculus of One Variable*

Learning Objectives

The aim of this course is to develop a conceptual understanding of the elementary theory of numbers and to expose the students to writing proper mathematical proofs.

Course Contents

Foundations: Principle of mathematical induction (with emphasis on writing a few basic proofs), binomial theorem, countable and uncountable sets, some basic results on countability, countability of Z, Q and uncountability of R.

Divisibility: Basic properties, division algorithm, GCD, LCM, properties of GCD, relation between GCD and LCM, Euclidean algorithm for finding GCD, Pythagorean triples, linear Diophantine equations, fundamental theorem of arithmetic, Euclid's lemma, existence of infinitely many primes.

Modular arithmetic: Basic properties of congruences, linear congruences, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

Number theoretic functions: Arithmetic functions (tau, sigma and Mobius) and their properties (specifically multiplicative property of the functions tau, sigma and the Mobius inversion formula), Euler's phi function and its properties, Euler's Theorem, Fermat's little theorem as a corollary of Euler's theorem.

Quadratic reciprocity: Primitive roots (order of an integer modulo n, primitive roots for primes), quadratic congruences, definition of quadratic residue, Legendre symbol and its properties, quadratic reciprocity law.

Continued fractions: Finite continued fractions, approximation of rational numbers by finite simple continued fractions, solution of linear Diophantine equations using finite continued fractions, infinite continued fractions, unique representation of irrationals as an infinite continued fraction, Pell's equation and its solutions using continued fractions.

Suggested Books

Textbooks

1. David Burton, Elementary Number Theory, 7th edition, McGraw Hill Education, 2012.
2. John Stillwell, Elements of Number Theory, 1st edition, Springer, 2003.

References

1. James Tattersall, Elementary Number Theory in Nine Chapters, 1st edition, Cambridge University Press, 1999.
2. Ya. Khinchin, Continued Fractions, 3rd edition, Dover, 1997.
3. Thomas Koshy, Elementary Number Theory with Applications, 2nd edition, Elsevier, 2007.

MTH 306: Ordinary Differential Equations (4)

Pre-requisites: *MTH 303 Real Analysis I*

Learning Objectives

This is the first course in the theory of Differential Equations. The aim of the course is to introduce students to the basic theory and problem-solving methods for first order and second order Ordinary Differential Equations.

Course Contents

First-Order Linear equations: exact equations, orthogonal trajectories, homogeneous equations, integrating factors, reduction of order

Second-order linear equations: equations with constant coefficients, method of undetermined coefficients, variation of parameters, power series solutions, special functions, applications

Higher-order linear equations

Some basic concepts of Fourier series

Quick review of elementary linear algebra, Picard's existence and uniqueness theorem, Sturm comparison theorem

Systems of first-order equations, homogeneous linear systems with constant coefficients

Non-linear equations: critical points and stability, Liapunov's direct method, Poincare-Bendixson theory

Suggested Books

1. George F. Simmons & Steven Krantz, *Differential equations*, Paperback edition, Tata-McGraw Hill 2009
2. G. Birkhoff & G. C. Rota, *Ordinary differential equations*, Paperback edition, John Wiley &Sons, 1989
3. E. Coddington & N. Levinson, *Theory of ordinary differential equations*, Paperback edition, Tata-McGrawa Hill, 2008
4. W. Hurewicz, *Lectures on ordinary differential equations*, Dover, New York, 1999

MTH 307/417: Programming and Data Structures

Learning Objectives

The main objective of the course is to introduce students to algorithmic and logical thinking, and the fundamentals of computer programming. The course includes some deeper aspects of the theory of computer science like effective data storage and retrieval techniques, sorting techniques etc. Since the course does not assume a prior knowledge in computer science, it will prove useful to students of all disciplines. The course is particularly relevant to students pursuing applied or computational sciences.

Course Contents

- Programming in a structured language such as C
- Data Structures: definition, operations, implementations and applications of basic data structures
- Array, stack, queue, dequeue, priority queue, double linked list, orthogonal list, binary tree and traversal algorithm, threaded binary tree, generalized list
- Binary search, Fibonacci search, binary search tree, height balance tree, heap, B-tree, digital search tree, hashing techniques

Suggested Textbooks

1. Donald E. Knuth, *The art of computer programming (*five volumes, 0 - 4*)*, Addison Wesley
2. V. Aho, J. E. Hopcroft & J. E. Ullman, *Data Structures & Algorithm*, Addison Wesley
3. W. Kernighan, D. M. Richie, *The C Programming Language*, Prentice Hall

MTH 308/412: Combinatorics and Graph Theory

Learning Objectives

Students will learn the basic combinatorial principles: inclusion-exclusion, multinomial coefficients and other counting arguments. They will also learn the combinatorial structures known as graphs and the associated concepts and ideas.

Course Contents

- *Combinatorics*: Elementary principles of combinatorics (permutations and combinations), binomial coefficients, inclusion-exclusion principle, generating functions, recurrence relation, pigeon-hole principle and Ramsey theory
- *Graph theory*: definition, isomorphisms, degree sequences, connectivity, trees, colourings, Eulerian graphs, directed graphs, network flows

Suggested Texts

1. R. A. Brualdi, *Introductory Combinatorics* (5^{th} Ed.), Prentice Hall
2. F. Harary, *Graph Theory*, Westview Press
3. Bondy, U. S. R. Murty, *Graph Theory* (1^{st} Ed.), Springer, GTM
4. S. M. Cioaba& M. Ram Murty, *A First Course in Graph Theory*, TRIM Series, HBA

MTH 311: Advanced Linear Algebra

Learning Objectives

This course reviews undergraduate linear algebra and proceeds to more advanced topics. Its purpose is to provide a solid understanding of linear algebra of the sort needed throughout graduate mathematics.

Course Contents

- *Linear transformations and Determinants*: The algebra of linear transformations, Multi-linear functions, The Grassman Ring
- *Elementary canonical forms*: Characteristic Values, Annihilating Polynomials, Invariant subspaces, Simultaneous Triangulation and Diagonalization, The Primary Decomposition Theorem, S-N Decomposition, Canonical forms and Differential Equations
- *The Rational and Jordan Forms*: Cyclic subspaces and annihilators, Cyclic decompositions and the Rational form, The Jordan Form, Computation of Invariant factors, Semi-simple operators
- *Operators on Inner product spaces*: Forms on Inner product spaces, Positive forms, Spectral Theory, Unitary Operators, Normal Operators
- *Bilinear forms*: Bilinear forms, Symmetric bilinear forms, Skew-symmetric

bilinear forms

Suggested Reading

1. K. Hoffman and R. Kunze, *Linear Algebra*, Prentice-Hall, 1961
2. Serge Lang, *Linear Algebra (2^{nd} Edition)*, Addition-Wesley Publishing, 1971
3. M.W. Hirsch and S. Smale, Differential equations, dynamical systems and linear algebra, Pure and Applied Mathematics, Vol. 60, Academic Press, 1974
4. P. Halmos, Finite dimensional vector spaces (2^{nd} Edition), Undergraduate texts in Mathematics, Springer-Verlag New York Inc., 1987
5. Serge Lang, Algebra, Graduate Texts in Mathematics (3^{rd} Edition), Springer-Verlag New York Inc., 2005

MTH 401: Fields and Galois Theory

Pre-requisites*: MTH 301*

Learning Objectives

Field Extensions are studied in an attempt to find a formula for the roots of polynomial equations, similar to the one that exists for a quadratic equation. The Galois group is introduced as a way to capture the symmetry between these roots; and the solvability of the Galois group determines if such a formula exists or not. In the 19th century, Galois proved that a formula does not exist for a general 5th degree equation. More importantly, the use of groups to study the symmetry of other objects is a pervasive theme in Mathematics, and this is traditionally the first place where one encounters it. The topics to be covered include irreducibility of polynomials, Field Extensions, Normal and Separable Extensions, Solvable Groups, and Solvability of polynomial equations by radicals, Finite fields, and Cyclotomic fields.

Course Contents

- Polynomial rings, Gauss lemma, Irreducibility criteria
- Definition of a field and basic examples, Field extensions
- Algebraic extensions and algebraic closures
- Straight Edge and compass constructions (optional)

- Splitting fields, Separable and Inseparable extensions
- Cyclotomic polynomials, Galois extensions
- Fundamental theorem of Galois theory
- Composite and Simple extensions, Abelian extensions over Q
- Galois groups of polynomials, Solvability of groups, Solvability of polynomials
- Computations of Galois groups over Q

Suggested Textbooks

1. Ian Stewart, Galois Theory (3rd Edition), Chapman & Hall/CRC Press (2004)
2. J. Rotman, Galois Theory (2nd Edition), Springer (2005)
3. D. J. H. Garling, A Course in Galois Theory, Cambridge University Press (1986)
4. D. S. Dummit and R. M. Foote, Abstract Algebra (2nd Edition), John Wiley & Sons (1999)

MTH 403: Real Analysis II

Pre-requisites*: MTH 303*

Learning Objectives

This course deals with the study of functions of several real variables and the geometry associated with such functions. There are two parts to this course. The first part deals with the study of differentiation and integration of such functions. The second part is devoted to the statement and proof of the higher dimensional version of the fundamental theorem of calculus, viz, Stoke's theorem (and its companions). This is one of the standard courses in any mathematics curriculum. It also serves as a first introduction to differential geometry and topology.

Course Contents

- Vector-valued functions, continuity, linear transformations, differentiation, total derivative, chain rule
- Determinants, Jacobian, implicit function theorem, inverse function theorem, rank theorem
- Partition of unity, Derivatives of higher order

- Riemann integration in R^n, differentiation of integrals, change of variables, Fubini's theorem
- Exterior algebra, simplices, chains of simplices, Stokes theorem, vector fields, divergence of a vector field, Divergence theorem, closed and exact forms, Poincare lemma

Suggested Reading

1. David Widder, Advanced Calculus, second edition, Dover, 1989
2. M. Spivak, Calculus on manifolds, fifth edition, Westview Press, 1971
3. J. Munkres, Analysis on manifolds, Westview Press, 1999.

MTH 404: Measure and Integration

Pre-requisites*: MTH 403 Real Analysis II*

Learning Objectives

The concept of `measure' generalizes the notion of length, area, and volume and Riemann integration of continuous functions, which have been studied in previous courses. In this course we provide the students with a solid background on the fundamentals of measure and integration theory and prepare them for advance courses in analysis and related areas.

Course Contents

- Topology of the real line, Borel, Hausdorff and Lebesgue measures on the real line, regularity properties, Cantor function
- σ-algebras, measure spaces, measurable functions, integrability, Fatou's lemma, Lebesgue's monotone convergence theorem, Lebesgue's dominated convergence theorem, Egoroff's theorem, Lusin's theorem, the dual space of C(**X**) for a compact, Hausdorff space, **X**
- Comparison with Riemann integral, improper integrals
- Lebesgue's theorem on differentiation of monotonic functions, functions of bounded variation, absolute continuity, differentiation of the integral, Vitali's covering lemma, fundamental theorem of calculus
- Holder's, inequality, Minkowski's inequality, convex functions, Jensen's inequality, $\mathbf{L}^p$ spaces, Riesz-Fischer theorem, dual of $\mathbf{L}^p$ spaces

Suggested Texts

1. W. Rudin, *Real and Complex Analysis*, third edition. Tata-McGraw Hill, 1987
2. H. Royden, *Real Analysis*, third edition, Prentice-Hall of India, 2008
3. R. Wheeden, A. Zygmund, *Measure and Integral*, Taylor and Francis, 1977
4. J. Kelley, T. Srinivasan, *Measure and Integral*, Volume I, Springer, 1987
5. Rana, *An Introduction to Measures and Integration*, Narosa Publishing House
6. E. Lieb, M. Loss, *Analysis*, Narosa Publishing House

MTH 405: Partial Differential Equations

Pre-requisites: *MTH 306 Ordinary Differential Equations*

Learning Objectives

This is an introductory course in partial differential equations for students majoring in mathematics. After discussing the solutions of first-order linear and quasi-linear equations in considerable detail we introduce the Cauchy problem for first and higher order equations and then briefly discuss the Cauchy-Kovalevski existence theorem and Holmgren's uniqueness theorem. We follow this by a study of second-order linear equations; here the goal is to understand the solutions of the three prototypical equations, Laplace, Wave and the Heat equation, in the classical set-up.

Course Contents

- *First-order equations*: linear and quasi-linear equations, general first-order equation for a function of two variables, Cauchy problem, envelopes
- *Higher-order equations*: Cauchy problem, characteristic manifolds, real analytic functions, Cauchy-Kovalevski theorem, Holmgren’s uniqueness theorem
- *Laplace equation*: Green’s identity, Fundamental solutions, Poisson’s equation, Maximum principle, Dirichlet problem, Green’s function, Poisson’s formula
- *Wave equation*: spherical means, Hadamard’s method, Duhamel’s principle, the general Cauchy problem
- *Heat equation*: initial-value problem, maximum principle, uniqueness, regularity

Suggested Texts

1. F. John, *Partial differential equations*, 4^{th} edition, Springer, 1982
2. G. B. Folland, *Introduction to Partial differential equations*, 2^{nd} edition, Princeton University Press, 1995
3. J. Rauch, *Partial differential equations*, Springer, GTM 128, 1991
4. L. Evans, *Partial differential equations,* American Mathematical Society GSM series, 1998

MTH 406: Differential Geometry of Curves and Surfaces

Pre-requisites*: MTH 306 Ordinary Differential Equations*

Learning Objectives

Curves and surfaces are the objects that are generalizations of the real line and the Euclidean plane respectively. The structures of the local geometry is lifted to these objects whereas, the global perspective changes. For example the parameters like curvature and torsion are introduced which are of no relevance in the real line and Euclidean plane. The course would mainly highlight these perspectives and we would classify the curves in terms of these parameters. Beyond these we would try to generalize them while we consider surfaces. Since the curvatures are non-trivial we would need technical notions of distance and metric over the surfaces and we will classify them using these notions. At the end, we will turn to some intrinsic connection of the geometry and topology of surfaces with these parameters (e.g. Gauss-Bonnet theorem).

Course Contents

- *Curves*: curves in space, tangent vector, arc length, curvature, torsion, Frenet formulas
- *Surfaces*: parametrization, tangent plane, orientability, first fundamental form, area, orientation, Gauss map, second fundamental form, Gauss curvature, ruled and minimal surfaces
- Geodesics, isometries of surfaces, Gauss' Theorema Egregium, Codazzi-Mainardi equations
- Gauss-Bonnet theorem for compact surfaces

Suggested Textbooks

1. Pressley, *Elementary Differential Geometry*, Springer, Indian reprint, 2004
2. Manfredo do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, 1976
3. D. J. Struik, *Lectures on Differential Geometry*, Dover, 1988
4. Barrett O'Neill, *Elementary Differential Geometry*, Second edition, Academic Press (Elsevier), 2006

MTH 407/607: Complex Analysis I

Pre-requisites: *MTH 303 Real Analysis I*

Learning Objectives

The learning objective of this course include the definition of analyticity, the Cauchy-Riemann equations and the concept of differentiability. Also to be learnt are the theorems on entire functions, residue theorem and applications and finally conformal mapping.

Course Contents

- *Complex numbers*: powers and roots, geometric representation, stereographic projection
- *Complex differentiability*: limits, continuity and differentiability, Cauchy Riemann equations, definition of a holomorphic function
- *Elementary functions*: sequences and series, complex exponential, trigonometric, and hyperbolic functions, the logarithm function, complex powers, Mobius transformations
- *Complex integration*: contour integrals, Cauchy's integral theorem in a disc, Cauchy's Integral Formula, Liouville's theorem, Fundamental Theorem of Algebra, Morera's theorem, Schwarz reflection principle
- *Series representation of analytic functions*: Taylor series, power series, zeros and singularities, Laurent decomposition, open mapping theorem, Maximum Principle
- *Residue theory*: residue formula, calculation of certain improper integrals, Riemann's theorem on removable singularities, Casorati Weierstrass theorem, the argument principle and Rouche's theorem

- *Conformal mappings*: conformal maps, Schwarz lemma and automorphisms of the disk and the upper half plane

Suggested Books

Texts

1. Elias M. Stein, Rami Shakarchi, *Complex Analysis (Princeton Lectures in Analysis),* Princeton University Press, 2003
2. Theodore W. Gamelin, *Complex Analysis*, Springer Verlag, 2001
3. John B. Conway, *Functions of one Complex Variable I*, Springer, 1978
4. E. Freitag and R.Busam, *Complex Analysis*, Springer, 2005

References

1. Lars Ahlfors, *Complex Analysis.* McGraw Hill, 1979
2. R. Remmert, *Theory of Complex Functions.* Springer Verlag, 1991
3. C. Caratheodory, *Theory of Functions of a complex variable*, AMS Chelsea, 2001

MTH 408/522: Numerical Analysis

Pre-requisites*: MTH 303 Real Analysis I*

Course Contents

- Round off errors and computer arithmetic
- *Interpolation*: Lagrange interpolation, divided differences, Hermite interpolation, splines, numerical differentiation, Richardson extrapolation
- *Numerical Integration*: trapezoidal, Simpsons, Newton-Cotes, Gauss quadrature, Romberg integration, multiple integrals
- *Solution of linear algebraic equations*: direct methods, Gauss elimination, pivoting, matrix factorizations
- *Iterative methods*: matrix norms, Jacobi and Gauss-Siedel methods, relaxation methods
- *Computation of eigenvalues and eigenvectors*: power method, householders method, QR algorithm

- *Numerical solutions of non-linear algebraic equations*: bisection, secan and Newton's, zeroes of polynomials

Suggested Textbooks

1. R. L. Burden, D. J. Faires, *Numerical Analysis*
2. E. K. Blum, *Numerical Analysis and Computation, Theory and Practice*, Dover, 2010
3. S. D. Conte, C. De Boor, *Elementary Numerical Analysis*, third edition, McGraw-Hill, 1980
4. D. M. Young, R. T. Gregory, *A Survey of Numerical Mathematics*, volumes 1 and 2, Dover, 1988

MTH 409: Optimization Techniques

Pre-requisites*: MTH 303 Real Analysis I*

Learning Objectives

The optimization algorithms deal with optimizing several real-valued functions with some constraints. We will study linear and non-linear programming techniques. The linear programming deals with optimizing linear functions with linear constraints using hyperplanes; whereas the non-linear programming deals with optimizing functions with constraints, possibly non-linear. A lot of real-world problems occurring in science and industry, which involves optimization, will be discussed in this course.

Course Contents

- Maxima and minima, Lagrange multipliers method, formulation of optimization problems, linear programming, non-linear programming, integer programming problems
- Convex sets, separating hyperplanes theorem, simplex method, two phase simplex method, duality theorem, zero-sum two-person games, branch and bound method of integer linear programming
- Dynamic programming, Bellman’s principle of optimality

Suggested Books:

1. Katta G. Murty, *Linear Programming*, Revised edition, Wiley, 1983

2. Griva, S. Nash, A. Sofer, *Linear and Non-linear Optimization*, second edition, SIAM, 2008
3. M. Bazaraa, H. Sherali, C. Shetty, *Non-linear Programming: Theory and Algorithms*, third edition, Wiley Inter-Science, 2006

MTH 410/514/620: Representation Theory

Pre-requisites*: MTH 301, MTH 302*

Learning Objectives

The aim of the course is to introduce representations of finite groups, their character theory, and some basic examples. Representation theory is used to study groups in various settings, including Physics and Chemistry.

Course Contents

- Representations of groups, subrepresentations, Irreducible representations, tensor product of representations, Maschke's theorem, Wedderburn decomposition
- Characters of representations, Generalized characters, Schurs lemma, Orthogonality, Regular representations, Decomposition theorems
- Representations of direct product of finite groups, Induced representations, Reciprocity theorem
- Representations and characters of standard finite and infinite groups: cyclic groups, dihedral groups, symmetric and alternating groups of small order etc.
- Applications of Representation Theory

Suggesting Books

1. J. P. Serre, *Linear Representations of Finite groups* (Graduate Texts in Mathematics), 2nd Edition, Springer-Verlag New York Inc., 1977
2. W. Fulton and J. Harris, *Representation Theory, A First Course*, 2^{nd} Indian reprint, Springer India, 2007
3. *G. James, M. Liebeck, Representations and Characters of Groups*, Cambridge University Press, 2001
4. M. Suzuki, *Group Theory II*, Springer-Verlag, 1983

MTH 411/511: Introduction to Lie Groups and Lie Algebras

Pre-requisites*: MTH 311: Advanced Linear Algebra*, *MTH 301*: Group Theory

Learning Objectives

The proposed course aims at providing and introduction to Lie groups, Lie algebras and their representations. The first part of the course focuses on matrix Lie groups (closed subgroups of GL(n; C)) and Lie algebras. The second part of the course deals with representations of semisimple Lie groups and Lie algebras. We begin with SU(2) and SU(3), as these cases very well illustrate the ideas of Cartan subalgebras, the roots, weights and the Wey1 group. We also look at Semisimple Lie groups and Lie algebras in general towards the end.

Course Contents

- *Matrix Lie Groups*: Definition and examples; Lie group homomorphisms and isomorphisms, Lie subgroups, polar decomposition.
- *Lie algebras*: matrix exponential and matrix logarithm, the Lie algebra of a matrix Lie group, Lie subalgebras, complexification of a real Lie algebra, Baker-Campbell-Hausdorff formula.
- *Representation Theory*: standard and adjoint representations, unitary representations, irreducible representations of su(2), direct sum ad tensor product of representations, dual representations, Schur's Lemma.
- *Semisimple Theory*: Representations of SU(3), weights and roots, the WeyI group. Semisimple Lie algebras, complete reducibility, Cartan subalgebras, root systems.

Suggested Books:

1. Hall, Brian Lie Groups, Lie Algebras, and Representations. Graduate Texts in Mathematics, Vol. 222, Springer Verlag, 2003.
2. Rossmann, Wulf. Lie Groups: An Introduction through Linear Groups. Oxford Graduate Texts in Mathematics 5, Oxford University Press, 2002.
3. Humphreys, James E. Introduction to Lie Alogebras and Representation Theory. Graduate Texts in Mathematics, Vol. 9, Springer, 1973.
4. Baker, Andrew. Matrix Groups: An Introduction to Lie Group Theory. Springer Veriag, 2002.

MTH 503/614: Functional Analysis

***Pre-requisites**: MTH 304 General Topology, MTH 404 Measure and Integration*

Learning Objectives

Functional analysis is the branch of mathematics concerned with the study of spaces of functions. This course is intended to introduce the student to the basic concepts and theorems of functional analysis with special emphasis on Hilbert and Banach Space Theory. This gives the basics for more advanced studies in modern Functional Analysis, in particular in Operator Algebra Theory and Banach Space Theory.

Course Contents

- Normed Linear spaces, Bounded Linear Operators, Banach Spaces, Finite dimensional spaces, Quotient Spaces
- Hilbert spaces, Riesz Representation Theorem, Orthonormal sets, Bessel's Inequality, Parseval's Identity, Fourier Series
- Dual Spaces, Dual of L^p spaces , Hahn-Banach Extension Theorem, Applications
- Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle
- Weak and Weak-* topologies, Hahn-Banach Separation Theorem, Alaoglu's Theorem, Reflexivity
- Compact Operators, Adjoint of an operator, Spectral theorem for Compact Self-Adjoint operators
- (If time permits) Banach Algebras, Ideals and Quotients, Gelfand-Mazur Theorem, Fredholm Alternative, Fredholm Operators, Atkinson's theorem

Suggested Books:

1. G.F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill, 2008
2. W. Rudin, Functional Analysis, McGraw Hill Book Company, 1973
3. J.B. Conway, A course in Functional analysis, GTM 96, Springer, 1990
4. F. Hirsch and G. Lacombe, Elements of Functional Analysis, GTM 192, Springer

5. S. Kesavan, Functional Analysis, TRIM 52, Hindustan Book Agency
6. Martin Schechter, Principles of Functional Analysis, Graduate Studies in Mathematics, American Math. Soc. 2nd Ed
7. B.V. Limaye, Functional Analysis, New Age books, 2nd Ed

MTH 504/604: Complex Analysis II

Pre-requisites*: MTH 407: Complex Analysis I*

Learning Objectives

The aim of this course is to introduce to some advanced topics of contemporary complex analysis. The course is intended for the students who have done a first course in complex analysis. The course will solidify the understandings of complex analysis and will prepare the students to use the concepts learned in this course to other areas of mathematics as well as in applied areas.

Course Contents

- Harmonic functions: mean value property, Schwarz reflection principle, the Poisson kernel, Dirichlet problem
- Maximum modulus principle, Hadamard three-circle theorem, Phragmen-Lindelof theorem, Rado's theorem
- Approximations by rational functions: Runge's theorem, Mittag-Leffler theorem, simply connected regions
- Space of analytic functions, Hurwitz theorem, Montel's theorem, space of meromorphic functions, proof of Riemann mapping theorem, analytic continuation along curves, statement of monodromy theorem
- Entire functions: infinite products, Weierstrass factorization theorem, gamma and zeta functions, little and big Picard theorems

Suggested Books

Texts

1. Conway J.B., Functions of One Complex Variable, Springer-Verlag NY, 1978
2. Rudin W., Real and Complex Analysis, McGraw-Hill, 2006
3. Lang, S., Complex Analysis, Springer, 2003.

4. Epstein B. and Hahn L-S., Classical Complex Analysis, Jones and Bartlett, 2011

References:

1. Carathodory C., Theory of functions of a complex variable, Vol. I and II, Chelsea Pub Co, NY 1954
2. Remmert R., Classical topics in complex function theory, Springer 1997
3. Ahlfors L., Complex Analysis, Lars Ahlfors, McGraw-Hill, 1979.

MTH 505/623: Introduction to Ergodic Theory

***Pre-requisites**: MTH 304 General Topology, MTH 404 Measure and Integration*

Learning Objectives

The word 'ergodic' is an amalgamation of the two Greek words 'ergon' (work) and 'odos' (path). Ergodic theory deals with the study of qualitative properties of flow (dynamical system) induced by actions of groups on spaces (measure spaces, topological spaces or manifolds). This is an introductory course where several fundamental examples of dynamical systems will be discussed. One of the main theorems, the Birkhoff's Ergodic theorem, which relates the time average of the flow with the space average, will be proved. This course will introduce necessary techniques and tools to understand a dynamical system. A good understanding of a first course in topology and measure theory is essential.

Course Contents

- *Discrete Dynamical systems*: definition and examples - maps on the circle, the doubling map, shifts of finite type, toralautomorphisms.
- *Topological and Symbolic dynamics*: transitivity, minimality, topological conjugacy and discrete spectrum, topological mixing, topological entropy, topological dynamical properties of shift spaces, circle maps and rotation number.
- *Ergodic Theory*: invariant measures and measure-preserving transformations, ergodicity, recurrence and ergodic theorems (Poincare recurrence, Kac's lemma, Von Neumann's ergodic theorem, Birkhoff's ergodic theorem), applications of the ergodic theorem (continued fractions, Borel normal numbers, Khintchine's recurrence theorem), ergodic measures for continuous

transformations and their existence, Weyl’s equidistribution theorem, mixing and spectral properties.

- Information and entropy - topological, measure-theoretic, and their relationship. Skew products, factors and natural extensions, induced transformations, suspensions and towers. Topological pressure and the variational principle, thermodynamic formalism and transfer operators, applications of thermodynamic formalism: (i) Bowen's formula for Hausdorff dimension, (ii) central limit theorems.

Suggested Books:

1. P. Walters, An Introduction to Ergodic Theory, Springer-Verlag, New York, 1982
2. M.G. Nadkarni, Basic Ergodic Theory, Second Edition, Hindustan Book Agency, India
3. M. Brin and G. Stuck, Introduction to Dynamical Systems, CUP, 2002
4. M. Pollicott and M. Yuri, Dynamical systems and Ergodic theory, CUP, 1998
5. P. R. Halmos, Lectures on Ergodic Theory, Chelsea, New York, 1956
6. W. Parry, B. Bollobas, W. Fulton, Topics in Ergodic Theory, CUP, 2004
7. A.B. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems, Cambridge, 1995

MTH 506/610: Fourier Analysis on the Real Line

Pre-requisites*: MTH 404 Measure and Integration, MTH 503 Functional Analysis: Normed linear spaces, completeness, Uniform boundedness principle, MTH 405 Partial Differential Equations: Basic knowledge of Laplacian, Heat and Wave equations*

Learning Objectives

This is an introductory course on Fourier Analysis on the real line. Fourier transform is a very useful tool to study various physical problems. In this course, we intend to rigorously study the key concepts of the subject. We will also study some applications of Fourier series (or Fourier integral) to partial differential equations and some number-theoretic problems.

Course Contents

1. The vibrating string, derivation and solution to the wave equation, The heat equation
2. Definition of Fourier series and Fourier coefficients, Uniqueness, Convolutions, good kernels, Cesaro/Abel means, Poisson Kernel and Dirichlet's problem in the unit disc
3. Mean-square convergence of Fourier Series, Riemann-Lebesgue Lemma, A continuous function with diverging Fourier Series
4. Applications of Fourier Series: The isoperimetric inequality, Weyl's equidistribution Theorem, A continuous nowhere-differentiable function, The heat equation on the circle
5. Schwartz space*, Distributions*, The Fourier transform on R: Elementary theory and definition, Fourier inversion, Plancherel formula, Poisson summation formula, Paley-Weiner Theorem*, Heisenberg Uncertainty principle, Heat kernels, Poisson Kernels
6. (If time permits/possible project topic) Definition of Fourier transform on Rd, Definition of X-ray transform in R2 and Radon transform in R3, Connection with Fourier Transform, Uniqueness

Suggested Books

Texts

1. E.M.Stein and R. Shakarchi, *Fourier Analysis: An Introduction, Princeton Univ Press*, 2003
2. (For topics marked with a *) W. Rudin, *Functional Analysis,* 2^{nd} Ed, Tata McGraw-Hill, 2006

References

1. J. Douandikoetxea, *Fourier Analysis* (Graduate Studies in Mathematics), AMS, 2000
2. L. Grafakos, *Classical Fourier Analysis* (Graduate Texts in Mathematics), 2^{nd} Ed, Springer, 2008

MTH 507: Introduction to Algebraic Topology

Pre-requisites*: MTH 301 Group Theory, MTH 304 General Topology*

Learning Objectives

This is a first course in algebraic topology. The subject revolves around finding and computing invariants associated with topological spaces. The first such invariant is the fundamental group of a pointed topological space which we'll study in detail along with the classification of covering spaces using fundamental group actions.

Course Contents

- *The Fundamental Group*: Homotopy, Fundamental Group, Introduction to Covering Spaces, The Fundamental Group of the circle $\mathbf{S}^1$, Retractions and fixed points, Application to the Fundamental Theorem of Algebra, The Borsuk-Ulam Theorem, Homotopy Equivalence and Deformation Retractions, Fundamental group of a product of spaces, and Fundamental group the torus $\mathbf{T}^2=\mathbf{S}^1\times\mathbf{S}^1$, n-sphere $\mathbf{S}^n$, and the real projective *n*-space $\mathbf{RP}^n$.
- *Van Kampen's Theorem*: Free Products of Groups, The Van Kampen Theorem, Fundamental Group of a Wedge of Circles, Definition and construction of Cell Complexes, Application to Van Kampen Theorem to Cell Complexes, Statement of the Classification Theorem for Surfaces, and Fundamental groups of the closed orientable and non-orientable surfaces of genus *g*.
- *Covering Spaces*: Universal Cover and its existence, Unique Lifting Property, Galois Correspondence of covering spaces and their Fundamental Groups, Representing Covering Spaces by Permutations – Deck Transformations, Group Actions, Covering Space Actions, Normal or Regular Covering Spaces, and Application of Covering Spaces to Cayley Complexes.

Suggested Books

1. J. R. Munkres, Topology (2nd Edition), Pearson Publishing Inc, 2000
2. Hatcher, Algebraic Topology, Cambridge University Press, 2002
3. M. A. Amstrong, Basic Topology, Springer International Edition, 2004
4. W. S. Massey, Algebraic Topology: An Introduction , Springer, 1977
5. J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988

6. M. J. Greenberg and J. R. Harper, Algebraic Topology: A First Course, Perseus Books Publishing, 1981
7. E. H. Spanier, Algebraic Topology, Springer, 1994

MTH 508/608: Introduction to Differentiable Manifolds and Lie Groups

***Pre-requisites**: MTH 303 Real Analysis I, MTH 304 General Topology, MTH 306 Ordinary Differential Equations, MTH 403 Real Analysis II*

Learning Objectives

This course aims to extend calculus from domains of Euclidean space to more general objects called differentiable manifolds, which are of fundamental importance in the study of higher geometry. These are higher dimensional analogues of curves and surfaces in space with which the students are familiar. This is an important course for students who want to pursue research in differential geometry, topology and related areas.

Course Contents

- *Differentiable manifolds*: definition and examples, differentiable functions, existence of partitions of unity, tangent vectors and tangent space at a point, tangent bundle, differential of a smooth map, inverse function theorem, implicit function theorem, immersions, submanifolds, submersions, Sard's theorem, Whitney embedding theorem

- *Vector fields*: vector fields, statement of the existence theorem for ordinary differential equations, one parameter and local one-parameter groups acting on a manifold, the Lie derivative and the Lie algebra of vector fields, distributions and the Frobenius theorem

- *Lie groups*: definition and examples, action of a Lie group on a manifold, definition of Lie algebra, the exponential map, Lie subgroups and closed subgroups, homogeneous manifolds: definition and examples

- *Tensor fields and differential forms*: cotangent vectors and the

  cotangent space at a point, cotangent bundle, covector fields or 1-forms on a manifold, tensors on a vector space, tensor product, symmetric and alternating tensors, the exterior algebra, tensor fields and differential forms on a manifold, the exterior algebra on a manifold

- *Integration*: orientation of a manifold, a quick review of Riemann integration in Euclidean spaces, differentiable simplex in a manifold, singular chains, integration of forms over singular chains in a manifold, manifolds with boundary, integration of n-forms over regular domains in an oriented manifold of dimension n, Stokes theorem, definition of de Rhamcohomology of a manifold, statement of de Rham theorem, Poincare lemma

Suggested Books

Texts

1. J. Lee, *Introduction to smooth manifolds*, Springer, 2002
2. W. Boothby, An Introduction to differentiable manifolds and Riemannian geometry, Academic Press, 2002
3. F. Warner, Foundations of differentiable manifolds and Lie groups, Springer, GTM 94, 1983
4. M. Spivak, A comprehensive introduction todifferential geometry, Vol. 1, Publish or Perish, 1999

References

1. G. de Rham, *Differentiable manifolds: forms, currents and harmonic forms*, Springer, 1984
2. V. Guillemin and A. Pollack., *Differential topology,* AMS Chelsea, 2010
3. J. Milnor, *Topology from the differentiable viewpoint,* Princeton University Press, 1997
4. J. Munkres, *Analysis on manifolds,* Westview Press, 1997
5. C. Chevalley, *Theory of Lie groups*, Princeton University Press, 1999
6. R. Abraham, J. Marsden, T. Ratiu, *Manifolds, tensor analysis, and applications*, Springer, 1988

MTH 509/609: Sturm-Liouville Theory

Pre-requisites*: MTH 306 Ordinary Differential Equations, MTH 404 Measure and Integration*

Course Contents

- *Fourier Series:* Fourier series of a periodic function, question of point-wise convergence of such a series, behavior of the Fourier series under the operation of differentiation and integration , sufficient conditions for uniform and absolute convergence of a Fourier series, Fourier series on intervals, examples of boundary value problems for the one dimensional heat and wave equations illustrating the use of Fourier series in solving them by separating variables, a brief discussion on Cesarosummability and Gibbs phenomenon

- *Orthogonal Expansions:* A quick review of L2 spaces on an interval, convergence, completeness, orthonormal systems, Bessel's inequality, Parseval's identity, dominated convergence theorem
- *Sturm-Liouville Systems:* linear differential operators, formal adjoint of a linear operator, Lagrange's identity, self-adjoint operators, regular and singular Sturm-Liouville systems, Sturm-Liouville series, Prufer substitution, Sturm comparison and oscillation theorems, eigenfunctions, Liouville normal form, distribution of eigenvalues, normalized eigenfunctions, Green's functions, completeness of eigenfunctions
- *Illustrative boundary value problems:* A technique to solve inhomogeneous equations using Sturm-Liouville expansions, one dimensional heat and wave equations with inhomogeneous boundary conditions, one dimensional inhomogeneous heat and wave equations, mixed boundary conditions, Dirichlet problem in a rectangle and a polar coordinate rectangle
- *Maximum Principle and applications:* maximum principle for linear, second-order, ordinary differential equations, generalized maximum principle for such equations, applications to initial and boundary value problems, the eigenvalue problem, an extension of the principle to non-linear equations
- *Orthogonal polynomials and their properties:* Legendre polynomials, Legendre equation, Legendre functions and spherical harmonics, Hermite polynomials, Hermite functions, Hermite equation, Laguerre polynomials, Laguerre equation, zeros of orthogonal polynomials on an interval, and a recurrence relation satisfied by them
- *Bessel Functions:* Bessel's equation, identities, asymptotics and zeros of Bessel functions

Suggested Books:

Texts:

1. Birkhoff, G &Rota G., *Ordinary Differential Equations*, John Wiley & Sons
2. Folland, G., *Fourier Analysis & Its Applications*, AMS
3. Protter, M. & Weinberger, H., *Maximum Principles in Differential Equations*, Springer

References:

1. Brown, J. & Churchill, R., *Fourier Series and Boundary Value Problems*, McGraw-Hill
2. Jackson, D., *Fourier Series and Orthogonal Polynomials*, Dover

MTH 510/615: Operator Theory and Operator Algebras

Pre-requisites: *MTH 503 Functional Analysis*

Learning Objectives

The goal of this course is to prove a far-reaching generalization of the spectral theorem for self-adjoint matrices. This is the classification of normal operators on a Hilbert space by their spectrum and multiplicity. Operator algebras provide the natural framework for the proof, and are a subject of vigorous research in modern functional analysis.

Course Contents

- Banach Algebras, Ideals, Quotients, homomorphisms, Unitization
- Invertible Elements, Spectrum, Gelfand-Mazur Theorem, Spectral Radius Formula
- Commutative Banach Algebras, The Gelfand Transform, Applications to Fourier Transforms, Weiner's Theorem, Stone-Weierstrass Theorem
- Compact and Fredholm Operators, Atkinson's Theorem, Index Theory
- C* algebras, uniqueness of the norm, Commutative C* algebras, Gelfand-Naimark theorem, Spectral Mapping theorem
- Functional Calculus, Positive Operators, Polar Decomposition
- Weak and Strong Operator Topologies, Von Neumann Algebras, Double Commutant Theorem

- Spectral measure, Spectral Theorem for Normal Operators, Borel Functional Calculus
- Multiplicity Theory, Abelian Von Neumann Algebras, Classification of normal operators upto unitary equivalence

Suggesting Books

1. G. J. Murphy, *C* Algebras and Operator Theory* (Academic Press Inc, 1990)
2. J. B. Conway, *A Course in Functional Analysis* (2nd Ed) (Springer, 1990)
3. R. G. Douglas, *Banach Algebra Techniques in Operator Theory* (2nd Ed) (Springer, 1998)
4. K. R. Davidson, *C* Algebras by Example* (Fields Institute Monograph, AMS 1996)
5. R. V. Kadison and J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras - Vol. I* (Academic Press Inc, 1983)
6. W. A. Arveson, *A Short Course in Spectral Theory* (Springer 2002)

MTH 512/612: Non-commutative Algebra

Pre-requisites*: MTH 301, MTH 302, MTH 401*

Course Contents

- Matrix Rings and PLIDs, Tensor Products of Matrix Algebras, Ring constructions using Regular Representation
- Basic notions for Noncommutative Rings, Structure of Hom(M,N), Semisimple Modules & Rings, the Wedderburn Structure Theorem, Simple Rings, Rings with Involution
- The Jacobson Radical and its properties, Primitive Rings and Ideals, Hopkins-Levitzki Theorem, Nakayama's Lemma , Radical of a Module, Local Rings, Chevalley-Jacobson Theorem, Kolchin's Theorem, Clifford Algebras.
- Prime and Semiprime rings, Rings of Fractions and Goldie's Theorems, Rings with ACC (ideals), Tensor Algebras, Algebras over large Fields, Deformations and Quantum Algebras.
- Hereditary Rings and their Modules, Division rings.

- Central Simple Algebras, Cyclic Algebras, Symbol Algebras, Crossed Products, the Brauer Group, the functor Br, the Skolem-Noether Theorem, the centralizer Theorem, calculation of Brauer group of commutative rings.

Suggesting Books:

1. L. Rowen, Graduate algebra: noncommutative view, Graduate Studies in Mathematics, 91.
2. B. Farb, R. Dennis, Noncommutative algebra, GTM, Springer-Verlag.
3. T. Y. Lam, A first course in noncommutative rings, GTM, Springer.
4. J. Golan and T. Head, Modules and the structure of rings: A primer, Pure and applied mathematics

MTH 513/613: Introduction to Riemannian Geometry

Pre-requisites: MTH 405 and MTH 508

Learning Objectives

A Riemannian manifold is a smooth manifold equipped with additional geometric structure called a Riemannian metric and this structure provides a framework to measure geometric quantities such as length and angles on the manifold. Associated with a Riemannian metric are the fundamental concepts of a Riemannian connection, geodesics and curvature. First, the basic properties and results associated to these are studied. The course then explores the relationship between geodesics and curvature. After studying such questions (which are local in nature), the focus turns to global questions and the course culminates in a study of certain important results concerning how curvature affects the topology of the manifold.

Course Contents

- *Review of differentiable manifolds*: vector bundles, tensors, vector fields, differential forms, Lie groups

- *Riemannian metrics*: Definition, examples, existence theorem; model spaces of Riemannian geometry

- *Connections*: connections on a vector bundle, linear connections, covariant derivative, parallel transport, geodesics

- *Riemannian connections and geodesics*: torsion tensor, Fundamental Theorem of Riemannian Geometry, geodesics of the model spaces, exponential map, convex neighborhoods, Riemannian distance function, first variation formula, Gauss' lemma, geodesics as locally minimizing curves; completeness, statement of Hopf-Rinow Theorem
- *Curvature*: Riemann Curvature Tensor, Bianchi identity, scalar, sectional and Ricci curvatures
- *Jacobi Fields*: Jacobi equation, conjugate points, second variation formula, spaces of constant curvature (if time permits)
- *Curvature and topology*: Gauss-Bonnet Theorem, Bonnet-Myers Theorem, Cartan-Hadamard Theorem

Suggesting Books

Texts:

1. J. M. Lee. Riemannian Manifolds, An introduction to Curvature. Graduate Texts in Mathematics. Springer (1997).
2. M. P. doCarmo. Riemannian Geometry. Birkhauser (1991).
3. S. Gallot, D. Hulin, J. Lafontaine. Riemannian Geometry. Springer (2004).

References:

1. I. Chavel. Riemannian geometry, a modern introduction. Cambridge University Press (2006)
2. S. Kobayashi, K. Nomizu. Foundations of differential geometry, vol. -I, Wiley Interscience Publication (1996).

MTH 516/616: Topology II

Pre-requisites *(Desirable): MTH 507 or MTH 605 and MTH 302 or MTH 601*

Learning Objectives

This is an advanced course in algebraic topology that builds on the concepts introduced in MTH 507 and MTH 605. Homology, which is the central topic of this course, finds applications in several areas of mathematical research, including low-dimensional topology and operator algebras. It is primarily meant for students who wish to pursue research in topology and related areas.

Course Contents

- *Simplicial Homology*: Simplicial Complexes, Barycentric Subdivision, and Simplicial Homology with examples
- *Singular and Cellular Homology*: Definition with examples, Homotopy Invariance, Exact Sequence of Relative Homology, Excision, Mayer-Vietoris Sequence, Degree of Maps, and Cellular Homology, Jordan-Brouwer Separation Theorem, Invariance of domain and dimension, Borsuk-Ulam Theorem, Lefschetz-Hopf Fixed Point Theorem, Axioms for homology, Fundamental group and homology, and Simplicial Approximation Theorem
- *Cohomology*: Universal Coefficient Theorem, Künneth Formula, Cup Product and the Cohomology Ring, Cap Product, Orientations on Manifolds, and Poincaré Duality
- *Higher Homotopy Groups*: Definition with examples, Aspherical Spaces, Relative Homotopy Groups, Long Exact Sequence of a triple, n-connected spaces, and Whitehead's Theorem

Suggesting Books:

1. A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2. E. H. Spanier, Algebraic Topology, Springer, 1994.
3. J. R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.
4. J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
5. M. J. Greenberg & J. R. Harper, Algebraic Topology: A First Course, Perseus Books Publishing, 1981.
6. W. S. Massey, A Basic Course in Algebraic Topology, Springer International Edition, 2007.
7. G. Bredon, Topology and Geometry, Springer International Edition.

MTH 517/617: Introduction to Algebraic Geometry

***Pre-requisites** (Desirable): MTH 301, MTH 302, and MTH 401*

Learning Objectives

This course aims to provide an introduction to some of the basic objects and techniques and objects of algebraic geometry with minimal prerequisites. The main emphasis is on geometrical ideas and so most of the treatment will be over algebraically closed fields. Results from commutative algebra will introduced and proved as required and so no prior experience with commutative algebra will be assumed. After introducing the basic objects and techniques, they will be illustrated by application to the theory of algebraic curves.

Course Contents

1. Closed subsets of affine space, coordinate rings, correspondence between ideals and closed subsets, affine varieties, regular maps, rational functions, Hilbert's nullstellensatz
2. Projective and quasi-projective varieties, regular and rational functions on projective varieties, products and maps of quasi-projective varieties
3. Dimension of varieties, examples and applications
4. Local ring of a point, tangent and cotangent space, local parameters, non-singular points and non-singular varieties
5. Birational maps, blowups, disingularization of curves
6. Intersection numbers for plane curves, divisors on curves, Bezout's theorem, Riemann-Roch theorem for curves, Residue theorem, Riemann-Hurwitz formula

Suggesting Books:

1. W. Fulton, Algebraic curves: An introduction to algebraic geometry, 2008 ed. (available online).
2. R. Shafarevich, Basic Algebraic Geometry, Vol. 1, Third Edition, Springer, Heidelberg, 2013.
3. S. Abhyankar, Algebraic geometry for scientists and engineers, Mathematical Surveys and Monographs 35, American Mathematical Society, 1990.
4. K. Smith et al, An invitation to algebraic geometry, Springer, 2004.

MTH 518/618: Commutative Algebra

Pre-requisites*: MTH 401 and its pre-requisites*

Learning Objectives

The aim of this course is to introduce commutative algebra. This theory has developed not just as a standalone area of algebra, but also as a tool to study other important branches of Mathematics including Algebraic Geometry and Algebraic Number Theory.

Course Contents

- Quotient Rings, Prime and Maximal ideals, units, Nilradical, Jacobson Radical, Operations on ideals, Extensions and contractions
- Tensor product of Algebras (only existence theorem), Rings and Modules of fractions, Local properties, Structure passing between R and $S^{-1}R$ (resp. M and $S^{-1}M$)
- Primary decompositions, Uniqueness theorems, Chain conditions, Noetherian and Artinian Rings, Lasker-Noether theorem, Hilbert basis theorem, Nakayama's lemma, Krull intersection theorem
- Integral dependence, Going up theorem, Integrally closed integral domains, Going down theorem
- Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals
- Valuations, Completions, Extensions of absolute values, residue field, Local fields, Ostrowski's theorem
- Hilbert's Nullstellensatz

Suggesting Books:

1. Introduction to Commutative Algebra, Atiyah, M and Macdonald, I.G., Levant Books, Kolkata
2. Graduate Algebra: Commutative View, Rowen, L.H., Graduate Studies in Mathematics, AMS
3. Commutative Algebra with a view towards Algebraic Geometry, Eisenbud, D., Springer

MTH 519/619: Introduction to Modular Forms

Pre-requisites *(Desirable): MTH 407: Complex Analysis I*

Learning Objectives

The aim of this course is to introduce the theory of modular forms with minimal prerequisites. The course is intended for the students who have done the standard courses in Linear Algebra and Complex Analysis. The results and techniques from these courses will be used to understand the space of modular forms and hence the students will solidify their understandings of some basic tools learned throughout mathematics. Numerous examples of modular forms will be given which are useful in solving some classical problems in number theory. The purpose is to make the modular form theory accessible without going into the advanced algebraically oriented treatments of the subject. At the same time this course introduces the topics that are at the forefront of the current research.

Course Contents

- The full modular group SL2(Z), Congruence subgroups, The upper half-plane H, Action of groups on H, Fundamental domains, The invariant metric on H
- Modular forms of integral weight of level one, Eisenstein series, The Ramanujan τ-function, Dedekind η-function, Poincare series, The valence formula and dimension formula, Modular forms of integral weight of higher level
- The Petersson inner product, Hecke operators, Oldforms and newforms, Dirichlet series associated to modular forms: Convergence, Analytic continuation, Functional equation
- (if time permits) Modular forms of half-integral weight: Definition and examples, Hecke operators, Shimura-Shintani correspondences between modular forms of integral weight and half-integral weight.

Suggesting Books:

1. M. Ram Murty, M. Dewar, H. Graves, Problems in the theory of modular forms, Institute of Mathematical Sciences - Lecture Notes 1, Hindustan Book Agency, 2015.
2. N. Koblitz, Introduction to elliptic curves and modular forms, Graduate Texts in Mathematics 97, Springer, 1993.
3. J. P. Serre, A course in arithmetic, Graduate Texts in Mathematics 7, Springer, 1973.
4. T. M. Apostol, Modular functions and Dirichlet series in number theory, GTM 41, Springer, 1990.
5. H. Iwaniec, Topics in classical automorphic forms, Graduate Studies in Mathematics 17, AMS, 1997.

6. F. Diamond and J. Shurman, A first course in modular forms, Graduate Texts in Mathematics 228, Springer, 2005.
7. T. Miyake, Modular forms, Springer Monographs in Mathematics, Springer, 2006.
8. G. Shimura, Modular forms: basics and beyond, Springer Monographs in Mathematics, Springer, 2012.

MTH 520/622: Introduction to Hyperbolic Geometry

Pre-requisites *(Desirable): MTH 304, MTH 407*

Learning Objectives

Hyperbolic geometry is arguably the most important area in modern geometry and topology. This course is intended to expose the student to the foundational concepts in hyperbolic geometry, and is specially tailored to prepare the student for advance topics in geometric topology.

Course Contents

- *The general Möbius group*: The extended complex plane (or the Riemman sphere) **C**; The general Möbius group Mob($\hat{\mathbf{C}}$); Identifying $\mathrm{Mob}^+(\hat{\mathbf{C}})$with the matrix group PGL(2; **C**); Classification of elements of elements of $\mathrm{Mob}^+(\hat{\mathbf{C}})$; Reflections and the general Möbius group Mob($\hat{\mathbf{C}}$); Conformality of elements in Mob($\hat{\mathbf{C}}$).
- *The upper-half plane model* $\boldsymbol{H^2}$: The upper half plane $\mathbf{H^2}$; The subgroup Mod($\mathbf{H^2}$); Transitivity properties of $\mathrm{Mob}^+(\mathbf{H^2})$; Geometry of the action of $\mathrm{Mob}^+(\mathbf{H^2})$; The metric in $\mathbf{H^2}$; Element of arc-length inH^2; Path metric in $\mathbf{H^2}$; The Poincaré metric d_H on $\mathbf{H^2}$; Geodesics in $\mathbf{H^2}$; Identifying the group $\mathrm{Mob}^+(\mathbf{H^2})$ of isometries of $(\mathbf{H^2}, \mathbf{d_H})$ with PSL(2; R); Ultraparallel lines in H^2.
- *The Poincaré disk model* ***D***: The Poincaré disk **D**; Transitioning from $\mathbf{H^2}$ to **D** via $\mathrm{Mob}^+(\mathbf{H^2})$; Element of arc-length and the metric $d_\mathbf{D}$ in **D**; The Group Mob(**D**) of isometries of $(\mathbf{D}, \mathbf{d_D})$; Centre, radii, and length of hyperbolic circles in **D**; Hyperbolic structures on holomorphic disks.
- *Properties of* $\boldsymbol{H^2}$: Curvature of $\mathbf{H^2}$; Convex subsets of $\mathbf{H^2}$; Hyperbolic polygons; Area of a subset of $\mathbf{H^2}$; Gauss-Bonnet formula - area of a hyperbolic triangle; Applications of Gauss-Bonnet Formula: Area of

reasonable hyperbolic polygons, existence of certain hyperbolic *n*-gons, hyperbolic dilations; Putting a hyperbolic structure on a surface using hyperbolic polygons; Hyperbolic trigonometry: triogometric identities, law of sines and cosines, Pythagorean theorem.

- *Non-planar models (if time permits)*: Hyperboloid model for the hyperbolic plane; Higher dimensional hyperbolic spaces.

Suggesting Books

1. James W. Anderson, *Hyperbolic Geometry* (2nd Edition), Springer, 2005.
2. Arlan Ramsay, Robert D. Richtmyer, *Introduction to Hyperbolic Geometry*, Springer, 1995.
3. Harold E. Wolfe, *Introduction to Non-Euclidean Geometry*, Dover, 2012
4. Alan F. Beardon, *The geometry of discrete groups* (Chapter 7), Springer, 1983.
5. Svetlana Katok, *Fuchsian Groups* (Chapter 1), Chicago Lectures in Mathematics, 1992.
6. John Stillwell, *Geometry of surfaces* (Chapter 4), Springer, 1992.

MTH 521/621: Introduction to Wavelets

Pre-requisites *(Desirable): MTH 311, MTH 404*

Learning Objectives

This is an introductory course on wavelet analysis. In this course we will introduce the basic notion of wavelets in different settings, namely for finite groups, discrete infinite groups and real line. This will provide the students an opportunity to know perspective applications of linear algebra and real analysis in mathematics and beyond.

Course Contents

- *Review of Linear Algebra*: Complex Series, Euler's Formula, Roots of Unity, Linear Transformations and Matrices, Change of Basis, diagonalization of Linear Transformations and Matrices, Inner Product, Orthogonal Bases, Unitary Matrices.
- *The Discrete Fourier Transform*: Definition and Basic Properties of Discrete Fourier Transform, Translation-Invariant Linear Transformations, The Fast Fourier Transform.

- *Wavelets on Finite Group Z_N*: Convolution on Z_N, Fourier Transform on Z_N, Definition of Wavelets and Basic Properties, Construction of Wavelets on Z_N.
- *Wavelets on Infinite Discrete Group Z*: Definition and Basic Properties of Hilbert spaces, Complete orthonormal Sets in Hilbert Spaces, The spaces $l_2(Z)$ and $L^2([-\pi, \pi))$, Basic Fourier Series, The Fourier Transform and Convolution on $l_2(Z)$ Wavelets on Z.
- *Wavelets on R*: Convolution and Approximate Identities, Fourier Transform on R, Bases for The Space $L^2(R)$, Belian-Low Theorem, Wavelets on R, Multiresolution Analysis, Construction of Wavelets from multiresolution Analysis, Construction of Compactly supported Wavelets, Haar Wavelets, Band-Limited Wavelets, Applications.

Suggesting Books

1. Michael W. Frazier: An Introduction to Wavelets Through Linear Algebra, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1999.
2. Eugenio Hernandez, Guido Weiss: A First Course on Wavelets, Studies in Advanced Mathematics. CRC Press, Boca Raton, FL, 1996.
3. Ingrid, Daubechies: Ten Lectures on Wavelets, CBMS-NSF Regional Conference Series in Applied Mathematics, 61. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992

MTH 601: Algebra I

Course Contents

Group Theory:

- Review of basics: Groups, subgroups, cyclic groups, quotient groups, Lagrange’s theorem and some applications, isomorphism theorems, composition series, Jordan-Holder theorem.

- Group actions: Definition and examples, Cauchy’s theorem, class equation, Sylow’s theorems and applications.

- Direct product: Definition and examples, structure theorem for finitely generated abelian groups, examples.

- Solubility: Derived and lower central series, soluble groups, examples.

- Free groups, group presentation, nilpotent groups (if time permits)

Rings and Modules:

- *Review of basics*: Rings, subrings, ideals, examples, ring homomorphism, quotient rings, isomorphism theorems, field, integral domain, prime and maximal ideals, characterization of prime and maximal ideals, direct product of rings, chinese remainder theorem.
- *Localization*: Definition and examples, universal property of localization, local ring, localization at a prime ideal.
- *Integral domains*: Euclidean domain, Principal ideal domain, Unique factorization domain, primes and irreducible elements, examples, Gauss's lemma, Eisenstein's criterion.
- *Polynomial rings*: polynomial rings in one and several variables, universal property, unique factorization property.
- *Basics on modules*: Modules, submodules, homomorphism of modules, isomorphism theorems, ring of endomorphisms of a module.
- *Structure theorems*: Direct product of modules, direct sum of modules, universal property of direct product and direct sum of modules, short exact sequence, short five lemma, structure theorem for finitely generated modules over a principal ideal domain (with proof), review the fundamental theorem of finitely generated abelian groups.
- *Canonical forms*: Rational and Jordan canonical forms.

Fields and Galois Theory:

- *Review of basics*: Fields, subfield, characteristic of a field, field homomorphism.
- *Field extensions*: Finite and algebraic extensions, splitting fields, normal extensions, algebraic closure, separable extensions, inseparable extensions, cyclotomic fields, finite fields.
- *Galois Theory*: Primitive element theorem, fundamental theorem of Galois theory, applications, simple extensions.

Suggesting Books:

1. Abstract Algebra, Dummit and Foote, Wiley Publications, 2nd edition.
2. Algebra, Hungerford, Springer Publications.
3. Algebra (3rd Edition), Serge Lang, Addison Welsey.
4. Basic Algebra, Jacobson, parts-I and II, Dover Publications Inc.; 2nd edition.
5. Algebra, Birkhoff and McLane, Chelsea Publishing Co.
6. A course in the theory of groups, D.J.S. Robinson, Springer; 2nd edition.

MTH 602: Algebra II

Course Contents

- Extension of rings, integral extensions, going up and going down theorems, integral closure, integral galois extensions
- Transcendental extension, transcendence basis, Noether normalization theorem, separable and regular extensions
- Algebraic varieties, Hilbert's Nullstellensatz, Spec of a ring
- Noetherian (and Artinian) rings and Modules
- Matrices and linear maps, determinants, duality, bilinear and quadratic forms
- Tensor product, basic properties, bimodules, Flat modules, extension of scalars, Algebras, Graded algebras, Tensor, symmetric and exterior algebras

Suggesting Books:

- S. Lang, Algebra, 3rd Edition, Addison Wesley.
- Jacobson, Basic Algebra I and II.
- Birkhoff and McLane, Algebra.
- Dummit and Foote, Abstract Algebra, 2nd Edition, Wiley.

MTH 603: Real Analysis

Course Contents

- *Several variable calculus*: A quick overview, the contraction mapping theorem, the inverse function theorem, the implicit function theorem.
- Riemann integration in R^n, $n \geq 1$.
- *Lebesgue measure and integration*: Measures, measurable functions, integration of nonnegative and complex functions, modes of convergence, convergence theorems, product measure, Fubini's theorem, convolution, integration in polar coordinates.
- Signed measures and differentiation, complex measures, total variation, absolute continuity, Fundamental theorem of calculus for Lebesgue integral, the Radon-Nikodym theorem and consequences.
- L^p spaces, the Hölder and Minkowski inequalities, Jensen's inequality, completeness, the Riesz representation theorem, dual of L^p spaces.

Suggested Books:

1. G.B. Folland, Real analysis: Modern techniques and their applications, 2nd Edition, Wiley.
2. W. Rudin, Principles of Mathematical Analysis, 3rd Edition, Tata McGraw-Hill.
3. W. Rudin, Real and Complex Analysis, 3rd Edition, Tata McGraw-Hill.
4. E.M. Stein and R. Shakarchi, Functional Analysis: Introduction to further topics in analysis, Princeton lectures in analysis.
5. T. Tao, Analysis I and II, 2nd Edition, TRIM Series 37, 38, Hindustan Book Agency.

MTH 605: Topology I

Course Contents

- *General Topology*: Connectedness, Compactness, Local Compactness, Paracompactness, Quotient Spaces, Topological Groups, and Baire Category Theorem.

- *Homotopy Theory*: Homotopy, Homotopy Equivalence and Deformation Retractions, Fundamental Group, Van Kampen Theorem, Deck Transformations, Group Actions, and Classification of covering spaces. Basic definitions of higher homotopy groups and long exact sequence of a bration.
- *Cellular and Simplicial Complexes*: Operations on Cell Complexes and Homotopy Extension Property. Simplicial Complexes - Barycentric Subdivision and Simplicial Approximation Theorem.

Suggesting Books

1. A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2. J. R. Munkres, Topology, Second Edition, Prentice Hall, 2011.
3. G. Bredon, Topology and Geometry, Springer International Edition, 2006.
4. W. S. Massey, A Basic Course in Algebraic Topology, Springer International Edition, 2007.
5. J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
6. J. R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.

MTH 606: Ordinary Differential Equations

Course Contents

- First-order equations
  - Direction fields, approximate solutions , the fundamental inequality, uniqueness and existence theorems, solutions of equations containing parameters
  - Comparison theorems
- Systems of first-order equations
  - Linear systems with constant coefficients: exponentials of linear operators, the fundamental theorem for linear systems, linear systems in the plane, canonical forms of linear operators on a complex vector space (S+N decomposition, nilpotent canonical forms, Jordan and real canonical forms), stability theory (saddle, spiral, and nodal points), phase portraits
  - Linear equations of higher order: fundamental systems, Wronskian, reduction of order, non-homogeneous linear systems, Green’s function

    - Non-linear systems: the fundamental existence-uniqueness theorem, dependence on initial conditions and parameters, the maximal interval of existence, the flow defined by a differential equation, linearization, the Stable Manifold theorem, the Hartman-Grobman theorem, stability theory of equilibria (saddles, nodes, foci and centres), Liapunov functions, La-Salle’s invariance principle, gradient systems

- Poincare-Bendixson theory: Limit sets, local sections, theorem of Poincare-Bendixson, Poincare's index, orbital stability of limit cycles, index of simple singularities

Suggesting Books

1. G. Birkhoff & G. C. Rota, Ordinary differential equations, Paperback edition, John Wiley & Sons, 1989
2. W. Hurewicz, Lectures on ordinary differential equations, Dover, New York, 1997
3. Morris Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, and Linear Algebra (Pure and Applied Mathematics (Academic Press), 1974
4. P. Hartman, Ordinary Differential Equations, New York, Wiley, 1964